## Problem 1.17

## Rolling drum

A drum of radius $R$ rolls down a slope without slipping. Its axis has acceleration $a$ parallel to the slope. What is the drum's angular acceleration $\alpha$ ?


## Solution

Because the drum rolls without slipping, the point of the drum in contact with the slope is instantaneously at rest. The reason for this is that vectorially the linear velocity due to rotation is equal and opposite to the linear velocity due to translation.

$$
R \omega=v
$$

Differentiate both sides with respect to $t$.

$$
\frac{d}{d t}(R \omega)=\frac{d v}{d t}
$$

The radius of the drum is constant and can be pulled in front of the derivative.

$$
R \frac{d \omega}{d t}=\frac{d v}{d t}
$$

The rate of change of angular velocity is the angular acceleration $\alpha$, and the rate of change of linear velocity is linear acceleration $a$.

$$
R \alpha=a
$$

Therefore,

$$
\alpha=\frac{a}{R} .
$$

